Lecture 3: Fundamentals Part 2 Pressurized Flow

WMD651: Water Resources Systems Design

2021 January 15

School of Environmental And Civil Engineering Technology, Seneca College

Johnathan D. Nault

Lecture Overview

- Last lecture covered fundamental concepts
- This lecture:
  - Pressurized v. open-channel flow
  - Head losses
  - Friction losses – Darcy-Weisbach, Hazen-Williams
  - Moody diagram
  - Local head losses – causes, representation
  - Pumps – principle, performance curves
  - System curves
  - Review of EGL for pumping pipeline

Quick Recap of Lesson 1

- What are our 3 key conservation laws?
  - Mass (continuity)
  - Energy
  - Momentum
- Energy grade line (EGL) components
- Hydraulic grade line (HGL) components
  - Velocity head, $V^2/2g$
- EGL v. HGL
  - When are they the same? If V = 0 m/s, EGL = HGL
  - When are they different? If V >> 0 m/s, EGL > HGL

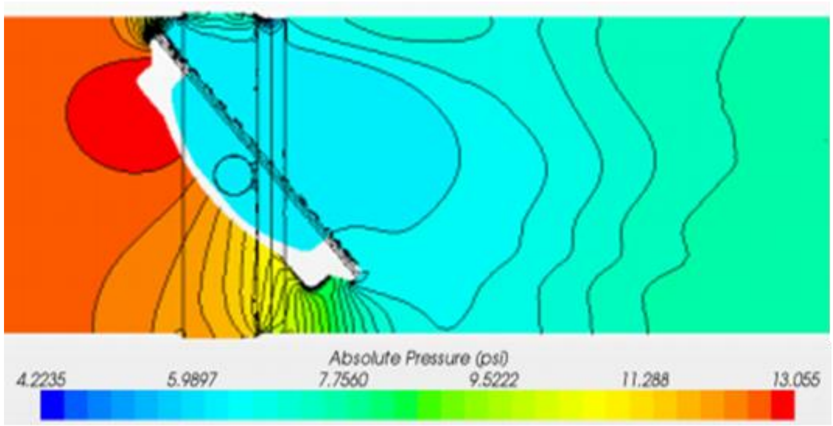
Pressurized and Open-Channel Flow

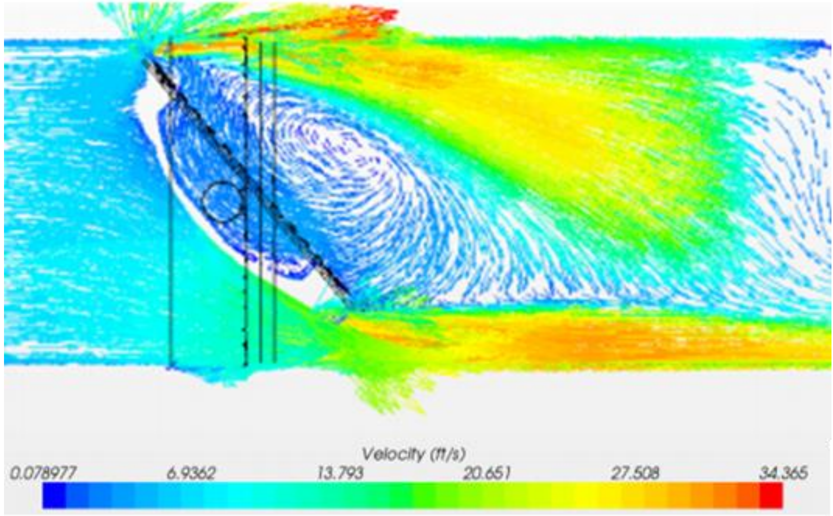
- **Pressurized flow:**
  - Conduit (pipe) is full, under pressure
  - HGL is above top elevation of pipe
  - Examples: watermains, sewage forcemains, submerged culverts
- **Open-channel flow:**
  - Water has a free surface that is open to air
  - HGL = water surface elevation
  - Examples: rivers, streams, sewers

Head Losses, an Uphill Battle

- Head losses are caused by viscous fluid resistance
- Always resist movement!
- Friction losses - $H_{friction}$:
  - Due to conduit wall friction
  - Distributed along pipes
- Local losses - H_{local}:
  - Due to high local velocities
  - Bends, valves, fittings
  - Concentrated at points
- Total head loss:

$$H_{total} = H_{friction} + H_{local}$$





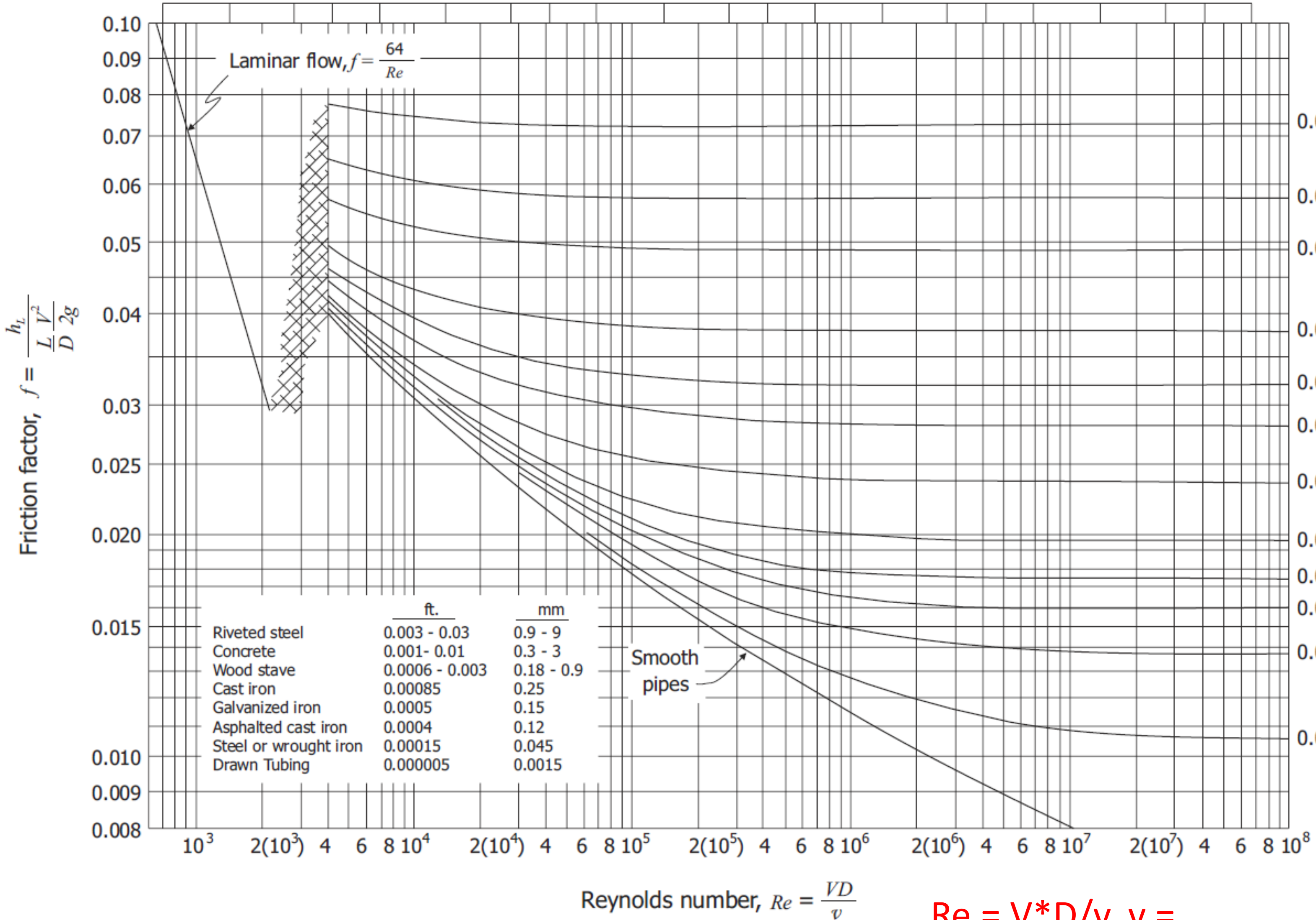
Friction Head Losses

- Friction losses depend on velocity (**flow**), pipe **roughness**, and pipe **length**
- Two commonly used friction loss equations
- Darcy-Weisbach (DW) equation:
  - Semi-theoretical formulation
  - For circular pipes: $$\boldsymbol{H_{friction} = f\left(\frac{L}{D}\right)\frac{V^2}{2g}}$$
  - f = friction factor (unit-less, typically 0.01 to 0.05)
  - L = pipe length (m)
  - D = pipe diameter (m)
  - V = flow velocity (m/s) → recall $V = Q/A$
  - g = 9.81 m/s^2

Darcy-Weisbach – Friction Factor

- DW friction factor (f) depends on:
  - Fluid properties – kinematic viscosity (ν)
  - Pipe wall roughness (ϵ)
  - Pipe diameter (D)
  - Reynolds number (Re)
- Reynolds number: $Re = VD/\nu$, unit-less
  - Characterizes flow regime (laminar, turbulent)
  - Laminar (smooth) flow for Re < 2000
  - Turbulent flow for Re > 4×10^3
- Determine f from Moody diagram

Darcy-Weisbach – Moody Diagram



	ft.	mm
Riveted steel	0.003 - 0.03	0.9 - 9
Concrete	0.001- 0.01	0.3 - 3
Wood stave	0.0006 - 0.003	0.18 - 0.9
Cast iron	0.00085	0.25
Galvanized iron	0.0005	0.15
Asphalted cast iron	0.0004	0.12
Steel or wrought iron	0.00015	0.045
Drawn Tubing	0.000005	0.0015

e/D

Re = V*D/v, v = kinematic viscosity

How to use the Moody Diagram

- **Step 1:** calculate $Re = VD/\nu$ (note: kinematic viscosity of water is ν = 1×10^{-6} m^2/s @ 20° C)

- **Step 2:** calculate relative roughness $B = \epsilon/D$

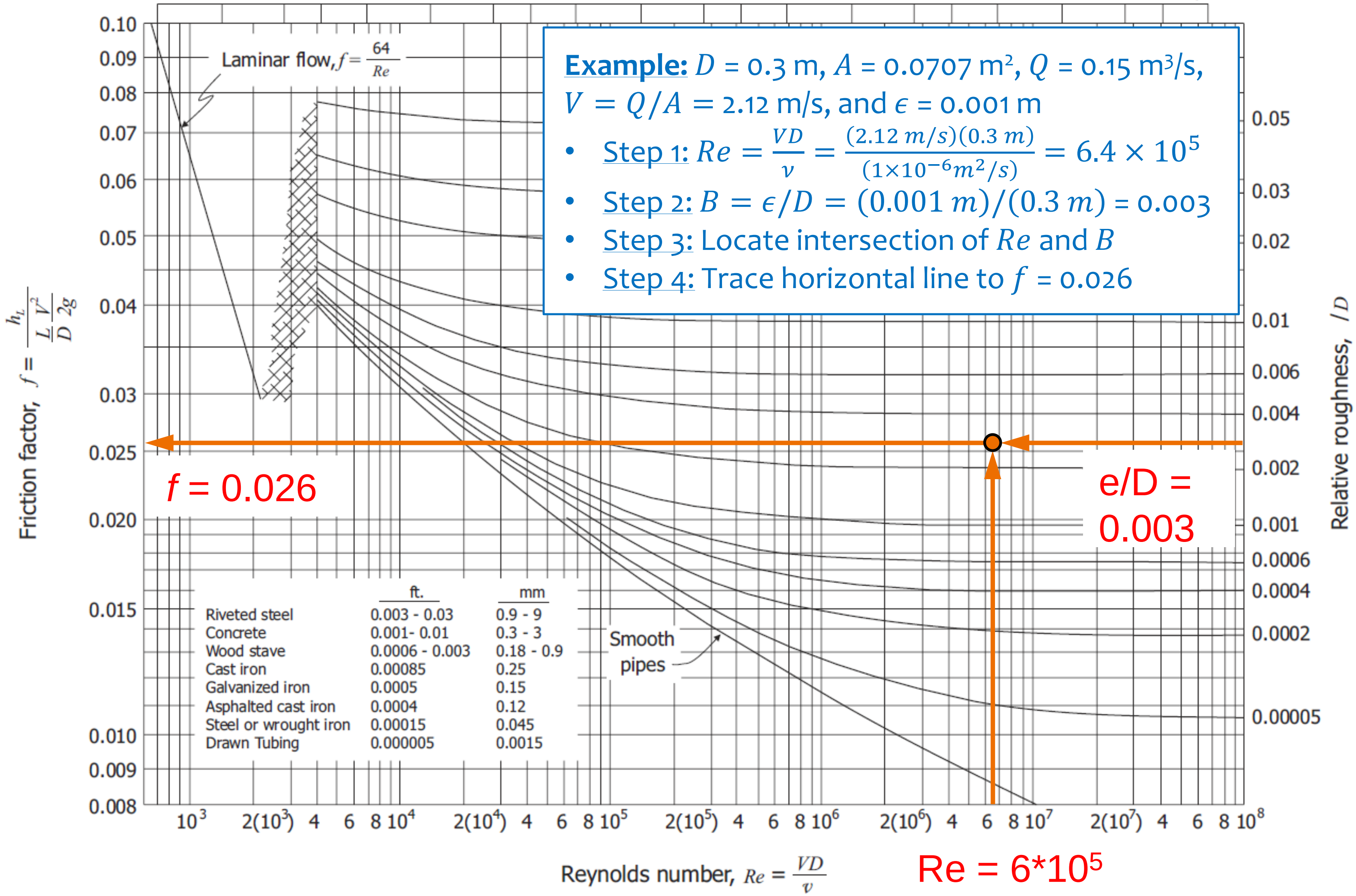
- **Step 3:** locate the intersection of Re and B on the Moody diagram

- **Step 4:** trace horizontal line left for f

Moody Diagram Example

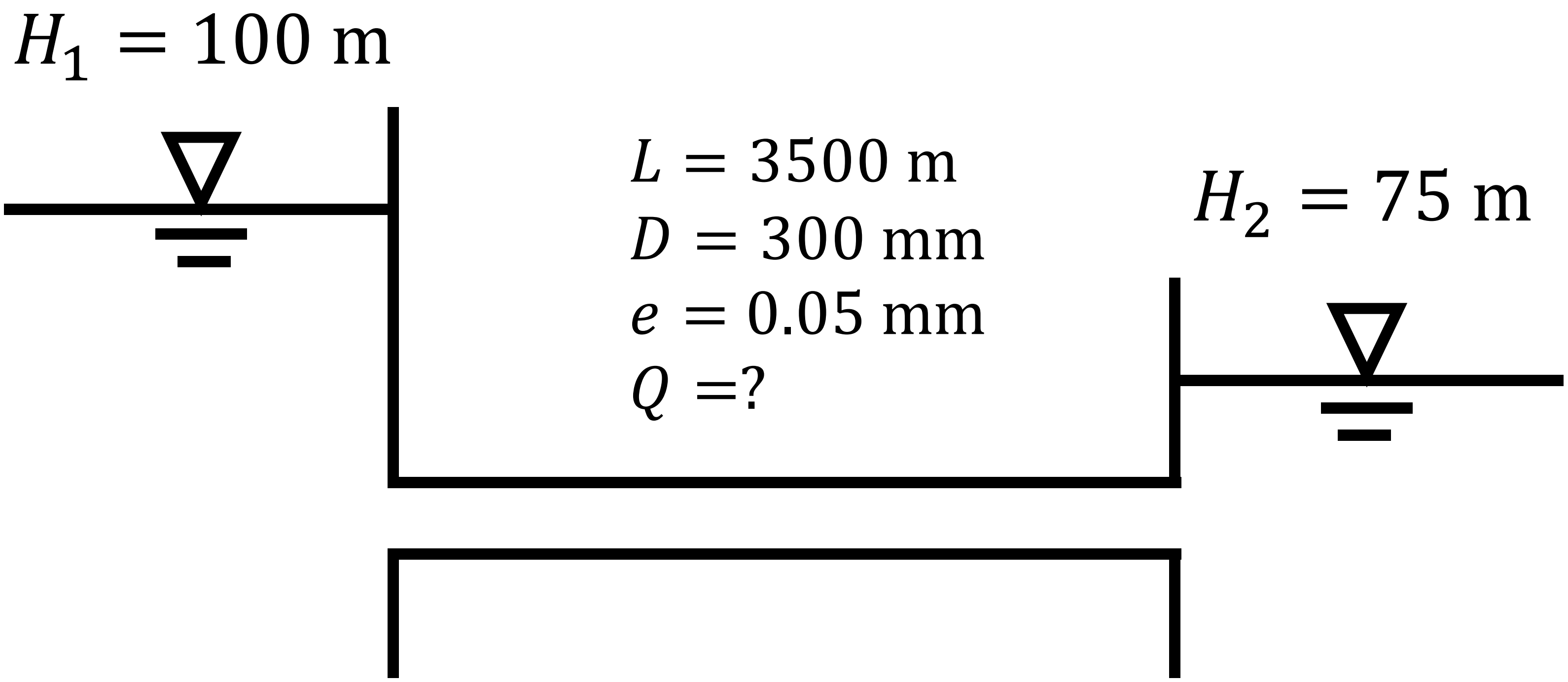
Example: D = 0.3 m, A = 0.0707 m², Q = 0.15 m³/s, $V = Q/A = $ 2.12 m/s, and ϵ = 0.001 m

- Step 1: $Re = \frac{VD}{\nu} = \frac{(2.12\ m/s)(0.3\ m)}{(1\times10^{-6} m^2/s)} = 6.4\times10^5$
- Step 2: $B = \epsilon/D = (0.001\ m)/(0.3\ m)$ = 0.003
- Step 3: Locate intersection of Re and B
- Step 4: Trace horizontal line to f = 0.026



Example – Darcy-Weisbach

Problem: determine the flow (Q) for the reservoir-reservoir system below.



Example – Darcy-Weisbach

Solution:

Solution approach:

1. Setup a system equation.
2. Estimate f_{trial} (typically start with f = 0.02) and solve for Q_{trial} from the system equation
3. Determine f_{final} from the Moody diagram using Q_{trial}, and solve for Q_{final} from the system equation
4. Check if $f_{trial} = f_{final}$. If not, repeat step 3 using f_{final} and Q_{final} until a solution is obtained.

Example – Darcy-Weisbach

Solution (continued):

First calculate a few solution constants:

- Pipe area: $A = \frac{1}{4}\pi D^2 = \frac{1}{4}\pi(0.3\ m)^2 = 0.0707\ m^2$
- Head loss equation:
  - $H_{friction} = f\ \left(\frac{L}{D}\right)\frac{V^2}{2g}$
  - From continuity, $V = Q/A \rightarrow V^2 = Q^2/A^2$
  - So: $H_{friction} = f\left(\frac{L}{2gDA^2}\right)Q^2$, where $L/2gDA^2$ is constant
  - $\frac{L}{2gDA^2} = \frac{(3500\ m)}{2(9.81\ m/s^2)(0.3\ m)(0.0707\ m^2)^2} = 119{,}000\ s^2/m^5$
- Relative roughness: $e/D = \frac{0.05\ mm}{300\ mm} = 1.7 \times 10^{-4}$

Example – Darcy-Weisbach

Solution (continued):

System equation:

- Energy equation: $E_1 = E_2 + H_{loss}$
  - $E_1 = H_1$ at the upstream reservoir (zero velocity)
  - $E_2 = H_2$ at the downstream reservoir (zero velocity)
- Updated energy equation:
  - $H_1 = H_2 + H_{loss}$
  - $H_1 = H_2 + f\left(\frac{L}{2gDA^2}\right)Q^2$
  - Rearrange: $H_1 - H_2 = f\left(\frac{L}{2gDA^2}\right)Q^2$
  - Isolate for flow: $Q = \sqrt{\frac{H_1 - H_2}{f\left(\frac{L}{2gDA^2}\right)}}$

Example – Darcy-Weisbach

Solution (continued):

Trial 1:

- Initial estimate of f_{trial} = 0.02
- $Q_{trial} = \sqrt{\dfrac{H_1 - H_2}{f_{trial}\left(\frac{L}{2gDA^2}\right)}} = \sqrt{\dfrac{100\ m - 75\ m}{0.02(119{,}000\ s^2/m^5)}}$ = 0.102 m^3/s
- $Re = \dfrac{VD}{\nu} = \dfrac{QD}{\nu A} = \dfrac{(0.3\ m)(0.102\ m^3 s)}{(1\times10^{-6}\ m^2/s)(0.0707\ m^2)}$ = 4.3×10^5
- Using Re and e/D, we can determine an updated friction factor from the Moody diagram
- Moody diagram: f_{final} = 0.0154, not converged → iterate
- *Note: Generally try to interpolate f to 3 decimals*

Example – Darcy-Weisbach

Solution (continued):

Trial 2:

- New estimate: f_{trial} = 0.0154
- $Q_{trial} = \sqrt{\dfrac{H_1 - H_2}{f_{trial}\left(\dfrac{L}{2gDA^2}\right)}} = \sqrt{\dfrac{100\ m - 75\ m}{0.0154(119{,}000\ s^2/m^5)}}$ = 0.117 m³/s
- $Re = \dfrac{QD}{\upsilon A} = \dfrac{(0.3\ m)(0.102\ m^3 s)}{(1\times 10^{-6}\ m^2/s)(0.0707\ m^2)}$ = 4.9×10^5
- Moody diagram: f_{final} = 0.0152 → converged

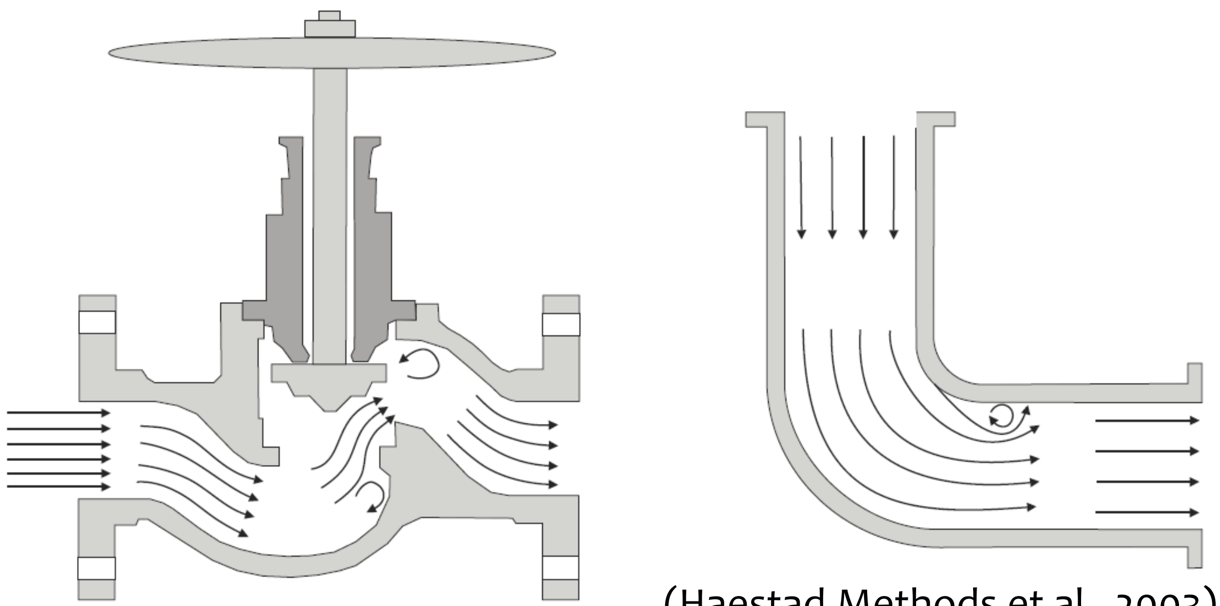
Thus, the system's flow is Q = 0.117 m³/s

Hazen-Williams Equation

- Hazen-Williams (HW) equation:
  - Empirical – based on observations from experiments
  - Only for water and circular pipes
  - Equation: $$H_{friction} = \frac{10.67 \cdot L}{D^{4.87}} \left(\frac{Q}{C}\right)^{1.852}$$
  - Q = flow (m³/s)
  - C = Hazen-Williams roughness (C-factor), typical values are 70 (rough) to 140 (smooth)
- Hazen-Williams C-factor is varies with:
  - Pipe diameter
  - Pipe material
  - Pipe roughness

Local Head Losses

- Unlike friction losses, local losses are concentrated
- Arise due to valves, bends, fittings (e.g., reducers)
- Equation: $H_{local} = k\frac{V^2}{2g}$, where k = a local loss coefficient that depends on the type of local loss



(Haestad Methods et al., 2003)

Sample Local Loss Coefficients

- Local loss coefficient (k) depends on the fitting
- Larger k indicate greater local losses
- Sample local loss coefficients:

Fitting	k
Conical contraction	0.05 - 0.08
Conical expansion	0.03 - 0.13
Gate valve	0.39
Globe valve	10
Angle valve	4.3
Butterfly valve	1.2

Fitting	k
90 deg. smooth bend	0.16 - 0.40
90 deg. mitered bend	0.8
Tee - line	0.30 - 0.40
Tee - branch	0.75 -1.8
Wye	0.30 - 0.50
Check valve	4

(Haestad Methods et al., 2003)

Generalized Head Loss Equation

- Darcy-Weisbach (DW), Hazen-Williams (HW), and local loss equations have similar structure
- DW:

$$H_{friction} = f\left(\frac{L}{D}\right)\frac{V^2}{2g} = \left(\frac{fL}{2gDA^2}\right)Q^2$$

- HW:

$$H_{friction} = \frac{10.67*L}{D^{4.87}}\left(\frac{Q}{C}\right)^{1.852} = \left(\frac{10.67L}{D^{4.87}C^{1.852}}\right)Q^{1.852}$$

- Local loss:

$$H_{local} = k\frac{V^2}{2g} = \left(\frac{k}{2gA^2}\right)Q^2$$

Generalized Head Loss Equation

- Head losses can be simplified into a single equation:
  - General equation: $\boldsymbol{H_{loss} = KQ^n}$
  - K = a resistance coefficient (units of [s^n/m^{2n-1}])
  - n = an exponent, depends on DW, HW, or local loss
  - Darcy-Weisbach: $\boldsymbol{K_{DW} = \frac{fL}{2gDA^2}}$ and $\boldsymbol{n = 2}$
  - Hazen-Williams: $\boldsymbol{K_{HW} = \frac{10.67 \cdot L}{D^{4.87} C^{1.852}}}$ and $\boldsymbol{n = 1.852}$
  - Local losses: $\boldsymbol{K_{local} = k/2gA^2}$ and $\boldsymbol{n = 2}$

Pumps, a source of energy!

- Unlike pipes and valves, pumps are an **energy source**
- Increase EGL
- Suction side E_1, discharge side E_2 → $E_2 > E_1$
- Pumps are used to overcome elevations and head losses

Suction:

$$E_1 = Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g}$$

Discharge:

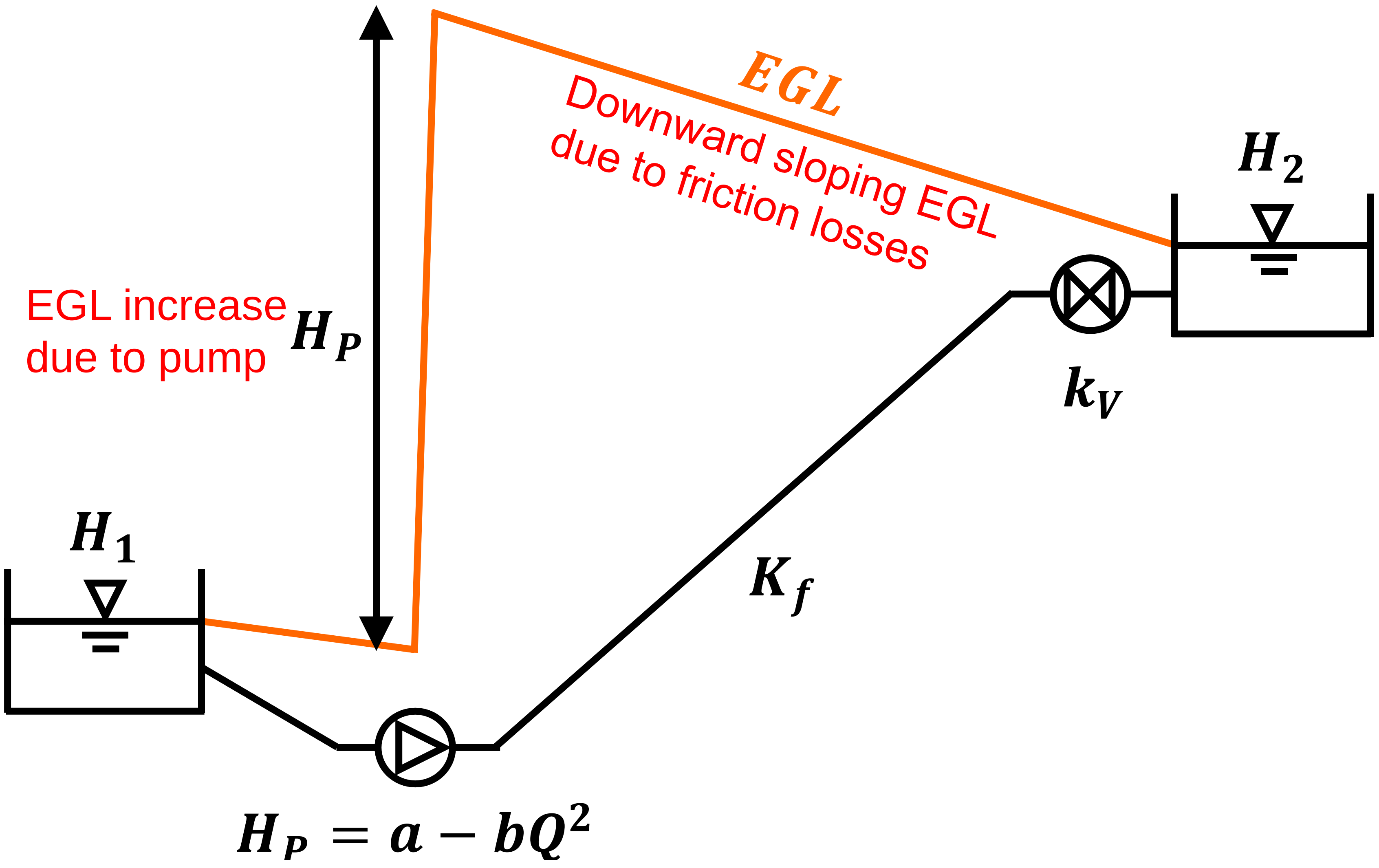
$$E_2 = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

E_1 →

→ E_2

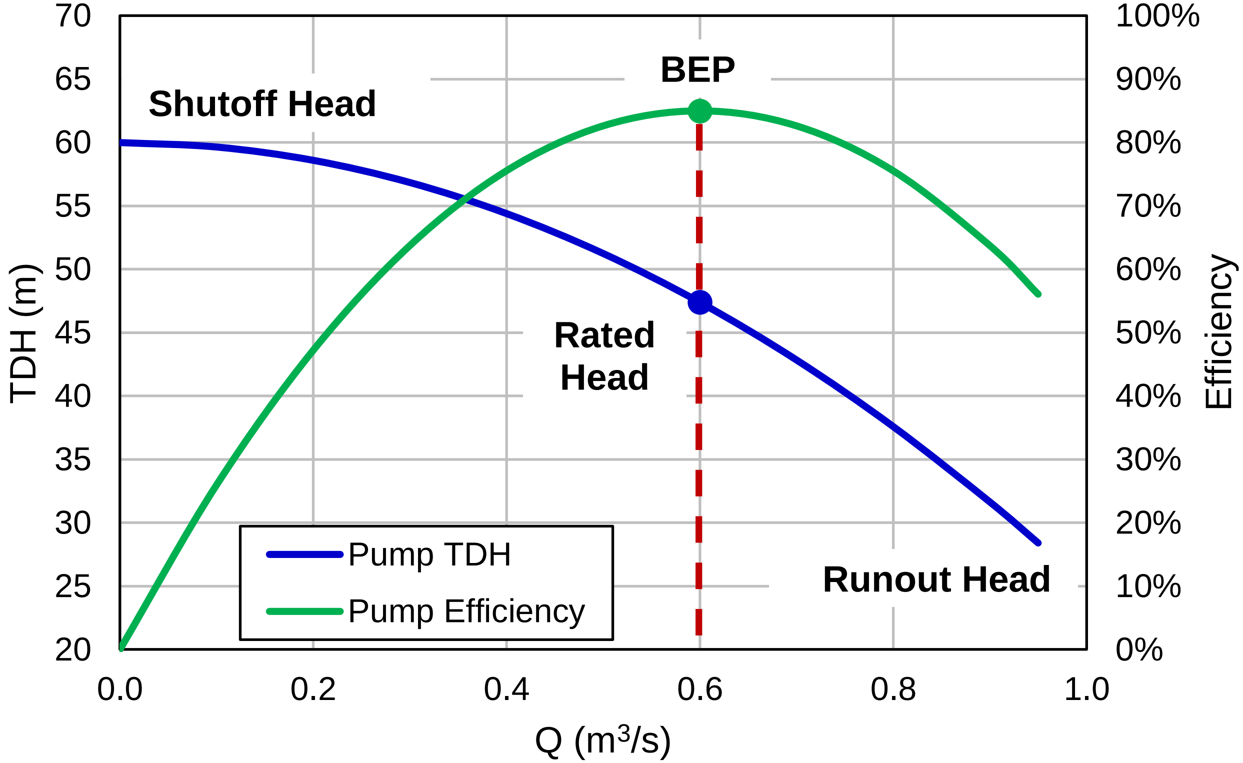
$$TDH = E_2 - E_1$$

EGL for a Pumping System



Pump Performance Curves

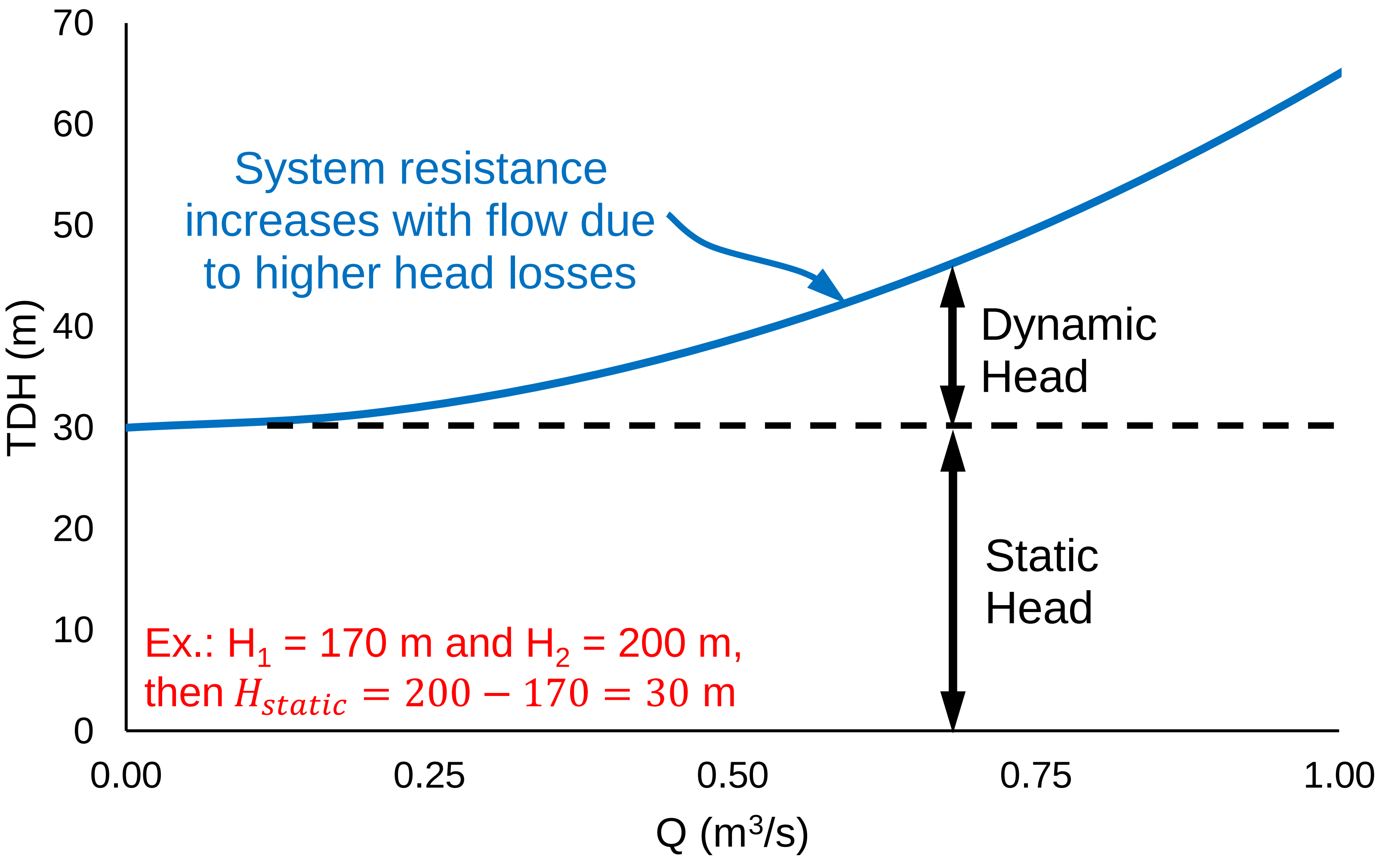
- Pump TDH and efficiency vary with flow
- Pumps are characterized by two sets of curves:
  1. Total dynamic head (TDH) v. flow, and
  2. Pump efficiency v. flow



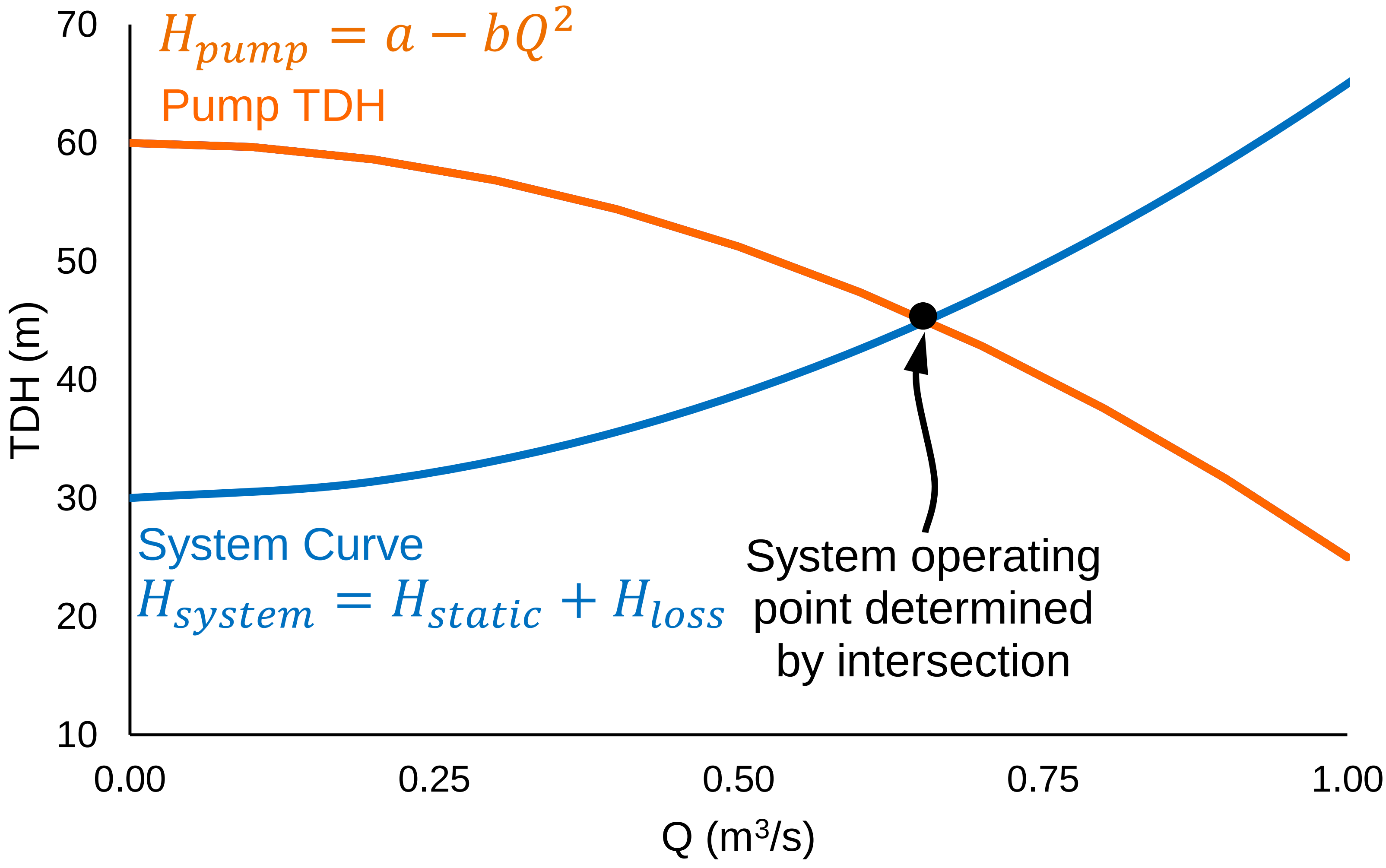
System Curves

- System resistance: $\boldsymbol{H_{system} = H_{static} + H_{loss}}$
- Static head component:
  - $\boldsymbol{H_{static} = H_2 - H_1}$
  - Represents the elevation difference between the start and end HGLs of system
  - Constant
- Dynamic head component:
  - $\boldsymbol{H_{loss} = H_{friction} + H_{local}}$
  - Represents head losses
  - Varies with flow (Q)
- Together, these form the system head

System Curve Plot



Pump and System Curve Plot



Summary